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ABSTRACT

This publication presents an assessment of various methods applicable for measurement of lender profitability with particular emphasis on net present value (NPV) in order to determine whether the Department of Education's current method for determining lender profitability within the Stafford Student Loan program is the best. The introduction points out that the NPV method of capital budgeting is currently accepted by economists and the financial community as the best of several methods within capital budgeting theory as it is easy to implement and determines the investment that maximizes the value of the firm. The body of the paper describes NPV in detail with the first subsection treating criteria for selection of this model. The second subsection describes NPV analysis by financial institutions covering revenues, expenses, cost of funds, and discount rate. A section on the weaknesses of this approach lists three drawbacks: errors in cash flow and discount rate if data are not carefully estimated, the same discount rate across loans and time periods implying that rates are not going to change over time; and difficulties accounting for every cost and benefit of student loans. Detailed appendixes treat ratio analysis, capital budgeting, portfolio theory, the capital asset pricing model, and the option pricing model. Included are 10 endnotes and a 27-item bibliography. (JB)

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I. EXECUTIVE SUMMARY

The objective of this task is to assess the various methodologies within financial theory applicable in the measurement of lender profitability. The primary method, Net Present Value (NPV), is discussed with respect to individual investments and to investments combined in portfolio. These are discussed generally in the body of the document and in detail in the appendices. Then the advantages and disadvantages of all accepted methods are discussed in the appendices. The NPV method is the current one used by the Department of Education and is the recommended method, as it is the only method where the results can be interpreted independently without using other methods.

II. INTRODUCTION

The Department of Education currently uses an analytic model that assesses lender profitability within the Stafford Student Loan program, across different borrower types. The model uses the net present value (NPV) method of capital budgeting, which is currently accepted by economists and the financial community as the best of several methods within capital budgeting theory. The reason for this is that the NPV method is easy to implement and determines the investment that maximizes the value of the firm. (See the discussion under section A. CRITERIA FOR SELECTION on p. 5.)

In addition to the capital budgeting methods, there are other recognized theories that can be used to analyze the profitability of student loans. These include Ratio Analysis, Portfolio Theory, Capital Asset Pricing Theory, and Option Pricing Theory. A discussion of each theory, including the various Capital Budgeting methods, is contained in APPENDIX A and includes an analysis of the advantages and disadvantages of each. A further, more technical appendix is included to more fully develop the Capital Asset Pricing Model and Portfolio Theory.

The NPV method is already used by the Department of Education for assessing individual loan profitability. The model that is used includes an approach that can take into account not only the profitability of individual loans through NPV but the profitability of all types of loans in combination.

III. NET PRESENT VALUE

The NPV as a capital budgeting method is the recommended method for several reasons:

1. The NPV uses data that is readily available to a financial institution. This data is utilized to estimate future revenues and expenses that are employed in the estimation of cash flows for the model. Also, it is used to estimate the discount rate from the cost of capital raised from all long-term sources, including long-term debt and equity. With other methods, data is not always readily available, and in some cases, the theories are not well developed enough to be able to adequately adapt them to Department of Education's needs.
2. The NPV method allows the decision-maker to choose the most profitable investment under all circumstances. Other methods may not be as reliable as the NPV method, since each has disadvantages that makes it unreliable under some circumstances, as discussed in the appendixes.
3. It is the only capital budgeting method that has all of the properties that make it the preferred method of evaluation of profitability. (See CRITERIA FOR SELECTION on p. 5.)

A. CRITERIA FOR SELECTION

As noted before, NPV is the only method of the capital budgeting methods that consistently demonstrates all of the necessary criteria (described below) for making reliable decisions. The best capital budgeting technique is easy to implement and chooses the investment that maximizes the value of the firm.

The following criteria are characteristics of the NPV model.

1. It considers all cash flows over the life of the loan.
2. It considers the time value of money. That is, the method should take into account that a dollar received today is worth more than a dollar received later. Therefore, each cash flow should be discounted from its time of receipt back to the present so as to be compared with the initial investment. The discount rate is an interest rate that is calculated to compare the opportunity cost of other possible investments with student loans.
3. NPV considers the best (most profitable) selection among all possible competing investments.
4. It considers all investments independently of each other. In other words, the method can be used to determine the profitability of an investment either on its own merits, or in comparison with other investments, thus offering flexibility.
5. Other methods, Payback Method, Internal Rate of Return (IRR), etc., can be used in conjunction with the NPV method. These other methods can satisfy different requirements of analyses, assuming that the NPV has been calculated and that the investment chosen is in accordance with this method. For example, the analyst may want to compare interest rates, but the NPV only gives a solution in dollar terms. Therefore, the IRR method could be used to satisfy this need. As discussed in appendix A, the IRR has drawbacks that the NPV method does not have and can lead to an incorrect choice of investment, but the NPV method can be used alone and is the preferred method.

B. NPV ANALYSIS BY FINANCIAL INSTITUTIONS

It is important for a financial institution to estimate the profitability of a loan or a portfolio of loans to ascertain whether funds should be invested in alternative assets that may return higher profits. The NPV method lends itself well to such an analysis as it gives a dollar amount of return that can be compared across assets to determine the best mix of assets. However, the components of the NPV equation must be estimated. These components are the revenues and expenses included in the cash flows and the discount rate.

1. REVENUES

In general, revenues for Stafford Student Loans are not difficult to estimate. They are paid quarterly by the Department of Education to the financial institution or paid by the borrower. The amount paid does fluctuate based on the level of the Treasury-bill rate, as the revenues are priced at a mark-up of 325 basis points (3 1/4 percent) above the Treasury-bill rate, so that some estimate of the future Treasury-bill rate must be determined.

An uncertainty inherent in these loans is the timing of the cash flows. These payments, in the form of delinquencies, can change cash flow patterns. In addition, borrowers can default on loans, at which point the default would become a prepayment, as the government would reimburse the institution for the amount of the loan.

2. EXPENSES

Expenses used in estimating cash flows include the interest expense on the deposits (cost of funds), as well as non-interest costs, such as the costs of originating the loans, the costs of collecting on the loans, and other costs. Financial institutions try to estimate these costs internally, but generally the smaller banks use estimated costs as an approximation from the Functional Cost Analysis published by the Federal Reserve.

Expense estimates on average are proportionally higher for small loans than for larger ones, as the processing costs are higher per dollar of loan. Loans that are too small may not generate sufficient income to cover the costs involved in origination and collection.

3. COST OF FUNDS

Many institutions use an internally-estimated cost of funds. This method allows institutions to adjust their loan rates to more closely match changes in costs. In practice, many financial institutions use the Functional Cost Analysis to estimate the cost of funds. However, larger lenders are likely to use other methods. Two methods used by financial institutions to estimate the cost of funds involve determining the average historical costs of funds and the marginal cost of funds. Average interest cost for all borrowed funds can be used, as can historical costs for a single source of funds, i.e., a 3-month CD. The main problem with using historical costs is that they provide no information concerning future costs. When interest rates rise, historical costs will understate the true cost.

Marginal cost of funds is used for both single-source funds and "pooled-source" funds that is a weighted average of the different sources of funds available to financial institutions.

When interest rates change, then the weighted marginal costs of funds should be more accurate than historical costs, as the marginal costs change with rates.

4. DISCOUNT RATE

The appropriate rate at which to discount the estimated cash flows must be calculated. Including short-term borrowing as a cost of capital is not correct, since it does not reflect the long-term nature of most funding needs by banks. To be most accurate, the discount rate should reflect the cost of all long-term capital, including debt and equity, used by a financial institution in funding loans.

This rate can be estimated in a variety of ways, some because of their simplicity, not their accuracy. Simple methods include using a mark-up over some base rate, e.g., Treasury-bill rate or the prime rate. This method was commonly used by banks in the 1970s and early 1980s.

The marginal cost of capital includes the cost of borrowing the next dollar with both long-term debt and equity. Marginal costs should reflect default risk even though the cost of default risk is low to nonexistent, given the government guarantee of loan performance to financial institutions. However, some cost of due-diligence may be included. In addition, marginal costs should reflect required returns to shareholders.

This discussion leads us to the next problem of estimation, the cost of equity (the return to shareholders). Generally, this is accomplished by estimating the cost of equity as a mark-up over the debt cost to compensate shareholders for their increased risk. This is a simple, but inaccurate, method to use. A more accurate method, the capital asset pricing model (CAPM), is more complicated, but can be adapted for use here (and is discussed in detail in the appendices).

By combining these marginal costs of debt and equity into a weighted average cost of capital, the institution can derive the closest approximation to the appropriate discount rate available.

C. WEAKNESSES OF NPV

The NPV method of evaluating the profitability is not perfect. There are several problems.

1. Estimation of future cash flows and of the discount rate can be erroneous if the data are not carefully estimated. In addition, the NPV does not measure the added risk to the entire portfolio of an additional loan. Since student loans have low-risk characteristics, then the inclusion of these loans in a portfolio of risky assets can reduce the overall risk of this risky

of risky assets can reduce the overall risk of this risky portfolio. Because this benefit will not be obvious in using the NPV to evaluate the student loans, profitability of the portfolio may even be underestimated.

2. Another weakness of the NPV method is that usually the same discount rate is used across all loans and even across all time periods. This use of a constant discount rate implies that rates are not going to change over time and that the risk on each loan is the same across borrowers. For the NPV to estimate the value of loans, the discount rate should be adjusted to reflect these differences. One adjustment might be to use a different discount rate estimated for different borrower types. Those at higher risk of default could be assigned a higher discount rate to reflect the added risk of default.

3. It is difficult to explicitly account for every cost and benefit of student loans in the cash flows. For example, the derived benefit of a bank's developing a continuing relationship with a borrower is not taken into account. Estimation of qualitative benefits or those later derived as directly attributable to the student loan is difficult to quantify for inclusion in cash flows. Cross-selling of services is an example of a benefit. Again, as stated above, the exclusion of these benefits of student loans from the profitability analysis may underestimate their value to the entire portfolio.

III. CONCLUSION

Much research and effort has been spent trying to determine the most effective method of profitability analysis and its component parts. So far, the preferred method (particularly by researchers) has been the NPV method, although each of the other methods has its own following, especially by practitioners. In reality, methods may be used in conjunction with others, so as to reflect different aspects of the profitability picture. Most important are the uses to which each model is put and the understanding by the users of the advantages and disadvantages of each.

In the analysis of the profitability of student loans, the NPV method is the best technique of the capital budgeting analyses, as described above. Cash flows can be estimated given the data available from financial institutions and from loan assumptions, and the cost of funds and the discount rate can be derived. Finally, since much of the data required in the other methods is either not available or not easily attainable, or has severe limitations, the NPV technique is the recommended one.

APPENDIX A

This appendix develops the theories of profitability analysis through a discussion of each technique and examines the advantages and disadvantages of each. Appendix B addresses, in more detail, the theoretical development of Portfolio Theory and the Capital Asset Pricing Model.

To evaluate the profitability of their financial services, financial institutions usually undertake two types of analyses: client profitability analysis (CPA) and loan-portfolio profitability analysis (LPP). Client profitability analysis is a method for measuring the return on a total client relationship and evaluating the return with the predetermined profit objective for that client. Loan portfolio profitability analysis applies a revenue and expenses approach to a category of loans, such as guaranteed student loans. The analysis takes all revenue (including origination fees, commitment fees, etc.) and all costs (including credit analysis costs, collection costs, and costs resulting from default, etc.) into account to estimate cash flows over the lives of the loans.

Our main objective is to evaluate loan portfolio profitability methods, and to assess the advantages and disadvantages of each method, and then to recommend the preferred method in the analysis of student loans. While student loans have many of the same revenue and cost elements common to other types of loans, they are unique in that they are guaranteed against default by the government, eliminating default costs. In addition, they have revenues set at a predetermined rate by the government at the Treasury-bill rate plus 325 basis points (3.25%) and they may be sold in the secondary market, thus effectively lowering the cost of holding them.

The method or methods chosen for analysis should be selected by how they are used. Use includes determining the profitability of student loans as a class of loans independent of other assets, or in comparison with other assets. Given this, the net present value (NPV) method would be the preferred method, as it gives a dollar value of profit and meets the necessary criteria. (See explanation below.) Sometimes, an analyst is given a set of results with which to compare his/her results. If ratio or some other method of comparison across assets is done, then the same analysis must be performed on student loans to be comparable. It is important that each method be understood for its shortcomings.

The methods include:

- Ratio Analysis
- Capital Budgeting Techniques
- Portfolio Theory
- Capital Asset Pricing Theory
- Option Pricing Theory.

Each method will be examined and evaluated in detail to determine its usefulness in estimating the profitability of student loans to a financial institution. However, it must be noted that no method is completely accurate. We can only use the data available to us and keep in mind the limitations of each method.

I. RATIO ANALYSIS

In using ratio analysis, many financial ratios can be calculated. However, we will discuss the two that are the most common.

In both CPA and LPP analyses, the rate of return on equity (ROE) measures net income per dollar of equity. The target after-tax return on equity (ATROE) as a measure of profitability of the bank is computed as follows:

(1) ATROE = (Loan Revenues - Loan Expenses)/Allocated Equity

where:

- o Loan revenues equal interest received on the loan plus loan handling, processing, and collection fees; Loan expenses equal cost of non-equity funds (deposits and borrowings) used by the borrower plus loan processing, servicing, and collection costs;
- o Allocated equity equals the overall equity-to-loan ratio for the bank times the amount of the loan balance outstanding. The numerator of this ratio can be adjusted to account for different types of loans, for example, discount loans, add-on interest loans, and loans with compensating balances.

The ATROE can be used with single-loan categories, as well as with the overall loan profitability of the bank, by changing allocated equity to reflect that portion of equity allocated to the type of loan analyzed or to total loans.

Another ratio that is used is return on assets (ROA) this is equal to net income divided by average total assets. This ratio measures net income per dollar of assets.

The ratio analysis of profitability provides information regarding historical performance measurement. It is of course helpful to evaluate the quality of previous credit (investment) decisions and to offer a guideline of necessary adjustment. However, it tells us nothing about the value and acceptability of perspective loan investments.

Both ROE and ROA are accounting measures of profitability, and therefore suffer several disadvantages common to other accounting measures. The main disadvantage includes the use of the balance sheet and income data that consist of historical data at book

value. Therefore, any profitability ratio that is calculated looks only at past decisions, not at current or future ones. The past is used to predict the future. Recognizing that this data is often all that is available to the analyst, the drawbacks are evident.

Since ratio analysis can add historical information, these measures can be used in conjunction with the other measures of profitability described below. However, this addition is at the discretion of the analyst given the use of the information and limitations of data.

II. CAPITAL BUDGETING

Capital budgeting is a theory used to determine if the net benefits (or cash inflows) derived over time are greater than the initial cost of the investment. These "cash flows" equal net income after income taxes. In each capital budgeting technique, except the modified internal rate of return, net cash inflows are revenue of costs that are incurred over the life of the investment. In any capital budgeting technique used, accurate estimation of cash flows is necessary to determine profitability of a potential investment.

> (In general) (s)ophisticated measures of return not only take the time value of money into account, but they also are developed from cash flows rather than from data developed by accrual accounting procedures. These procedures introduce arbitrary allocations among periods which are not needed for appraising projects and which do cause erratic fluctuations in the measures of return based on accounting data. For a short-lived project, the difference between simple and time adjusted rates of return may be of little significance. However, differences between rates of return based on cash flows and accounting figures may be substantial. Therefore, the use of measures of return calculated from cash flows rather than accrual accounting data may be important in accurate evaluation of investment proposals.[1]

A. CASH FLOW ESTIMATION

The cash flows of potential loans have to be estimated subjectively by loan officers and incrementally based on additions to costs and revenues attributable to the investment. However, this does not mean that the estimation is arbitrary. The distribution of cash flows can be inferred from the historical data of a bank or its peer banks if the distribution is stable and stationary. The accuracy of estimation can be enhanced with the help of standard cost estimation from sources, e.g., the Federal Reserve's Functional Cost Analysis (FCA) system. The program provides a bank with information on the income, expenses, and net earnings of specific operating functions. The data are reported

for three groups of institutions: those with deposits less than $50 million, deposits $50-$200 million, and deposits greater than $200 million. This enables bankers to compare their costs and revenues to those of other institutions of similar size and deposit structure. In addition, to be consistent, cash flows must be estimated on an after-tax basis, since the correct assumption is that the investment is funded with after-tax dollars, either through debt or equity.

B. UNCERTAINTY OF CASH FLOWS

Careful estimation of cash flows is critical to profitability analysis in capital budgeting. An important assumption in the following discussion is that the cash flows throughout the life of an investment project are 1) known with certainty, 2) are of equal lives, and 3) are of approximately equal risk. These assumptions are necessary unless adjustments have been made to the cash flows to take these three factors explicitly into account. In the real world, however, cash flows are affected by general economic conditions, industry factors, and corporate characteristics. As we have no way to predict what will happen to the economy, to industries, or financial institutions, the cash flows of a project at best are known with uncertainty. However, adjustments can be made that increase the accuracy of the cash flow estimates.

1. INCORPORATING INFLATION

One of the primary uncertainties in capital budgeting involves the estimation of inflation and its effects on cash flows. The analyst may wish to incorporate anticipated inflation into each cost and benefit. This incorporation may assume that inflation affects all cash flows equally or that costs are affected in a different way than are revenues. Of course, the latter assumption adds to the difficulty of the analysis.

Cash flows are used in the several methods of capital budgeting, discussed in the following section. In the discounting of cash flows, the interest rate[2] used is a nominal rate that already includes anticipated inflation. Therefore, for the analysis to be consistent, the cash flows must also be adjusted for anticipated inflation. This can be done by anticipating inflation and adjusting the cash flows accordingly. It is critical that any forecast of inflation be similar to the one already incorporated in the interest rate and that it be incorporated in a consistent manner across all cash flows.

In the analysis of student loans, however, lenders are at least partially protected from changes in inflation, as cash inflows are adjusted over the life of the loan to reflect changes in inflation as it is incorporated in interest rates. That is, as inflation rises, interest rates rise commensurately, albeit with a

lag. Therefore, as lenders receive compensation from the government and/or the borrower based on the Treasury-Bill rate plus a margin, then cash inflows are automatically inflation-adjusted. Correspondingly, cash outflows attributed to the investment beyond the initial cost must be adjusted for inflation. Whether the adjustment assumes equal effects across all cash flows or assumes different effects on inflows versus outflows is a judgement call by the analyst. However, the assumptions made should be explicitly stated.

2. ADJUSTMENTS FOR INVESTMENT LIFE

In comparing investments, the estimated life of each loan or type of loan must be equal for accurate evaluation. If the first investment has an expected life of three years and the second has an expected life of nine years, then there is no consistent way to determine which investment is better, unless adjustments are made to the lives of the investments to project a common denominator. One method for doing this is to make the first investment equal to three consecutive investments so that it equates to the one nine-year investment. If, however, the lives of the investments are not so easily matched, say the lives are five and eleven years, respectively, then a second method may be used. This second method is the equivalent annual annuity method. It involves finding an investment's NPV over its initial life and projecting all cash flows as if they continued forever (in perpetuity). Then these cash flows are discounted using the appropriate interest rate.

There are three serious weaknesses in estimating the lives of investments. First, if inflation is expected to rise, then costs and benefits will change, but the cash flows, unless adjusted, will be static over time. Second, over time, new technology will change the cash flows. Third, estimating the lives of investments is at best a good estimation. With a bank's estimation in loans, the estimation is probably fairly accurate, but can be altered by prepayments and defaults. In actual analysis, an analyst would probably not be too concerned with differences in lives of only one to two years.

3. ADJUSTMENT FOR RISK

In general, capital budgeting techniques are project specific. However, consideration must be given to the effect of risk regarding all investments of a bank. When trying to determine the risk of a given project on the bank, the discount rate is adjusted to compensate for any perceived added risk. An investment with less perceived risk than that average to the bank would be given a lower discount rate than would be given to a higher-risk investment. Therefore, by varying the discount rate, the capital-budgeting techniques can help the analyst estimate the return to the bank of an investment, while accounting for any added risk.

C. CAPITAL BUDGETING TECHNIQUES

The best capital budgeting technique is defined as the method that is easy to implement and selects the set of projects, or investments, that maximizes the institution's value, and hence, its shareholders' wealth.

For a capital budgeting method to consistently lead to correct investment decisions, it must possess the following properties:

1) All cash flows throughout the entire life of a project should be considered.
2) The time value of money should be considered, that is, a dollar today is more valuable than a dollar in the future. The cash flows should be discounted at the opportunity cost of funds. This means that cash flows received in the future are adjusted to be equal to today's value, assuming that a sum of money today can be invested at a given interest rate that over time would return interest payments equal to the same cash flows calculated above. The opportunity cost represents competing, alternative interest rates at which one can invest.
3) When selecting among a set of mutually exclusive projects (choose one and not the other), the method must choose that project that maximizes the institution's stock price (or shareholder's wealth).
4) The value additivity principle must be obeyed. In other words, managers should be able to consider one project independently of others and evaluate its merits without looking at it in relationship to others.

There are generally eight methods of capital budgeting used. Each method reflects the above properties to various degrees. The various alternative methods of capital budgeting, are discussed in the following pages. Three of these methods do not consider the time value of money. That is, the cash flows are not discounted. The other four techniques do discount cash flows. The following analysis is divided along these lines.

1. NON-DISCOUNTED CASH FLOW METHODS

a. PAYBACK PERIOD

The payback period method is simply the estimated number of years required to recover the original investment in a project or investment.

The advantages to payback are that it is simple to compute, easy to understand, and is a rough measure of risk (mainly liquidity risk), as short payback periods can be considered less risky than long payback periods. Long-term projects are subject to greater estimation errors of cash flows.

Payback provides information on how long the initial investment will be tied up. When there is a loan repayment commitment, this measure is important, as it can indicate when a bank is expected to recover its initial outlay in time to make another investment or payment of debt.

However, there are many disadvantages. This method does not consider cash flows beyond the payback period, the timing of the cash flows within the payback period, or the time value of money. (It does not use discounted cash flows.) In addition, it offers no information on how much the project contributes to the wealth of shareholders.

Although frequently used, the payback measure should not be used as the only measure of investment profitability.

b. ACCOUNTING RATE OF RETURN (Return on Investment, ROI)

1. SIMPLE RATE OF RETURN is equal to average after-tax profit divided by the initial cash outlay. This method is commonly used as a measure of project profitability. Its one advantage is that it does consider earnings after the payback period.

The disadvantages, on the other hand, are many. First, the measure does not consider the time value of money or the timing of income and total earnings generated during the project's life. Second, it does not adequately differentiate between early returns on investments and later returns. And third, it uses the accounting measure of profitability instead of cash flows. This decision leads to the maximization of the accounting profit rate, and hence, is inconsistent with the objective to maximize share price, which is the discounted value of future cash flows.

2. CASH RATE OF RETURN is equal to average annual net cash flow divided by the original investment.

Although the two measures are essentially the same, the cash rate of return may be a better measure of project profitability than the simple rate of return method when cash returns are used for investment. This measure approximates the discounted cash-flow methods, but fails to consider differences in the timing of cash flows.

2. DISCOUNTED CASH FLOWS

a. DISCOUNTED PAYBACK PERIOD (Present value payback)

This method estimates the expected cash flows over the life of an investment and estimates the number of years required to recover the investment from these discounted cash flows.

Advantages and disadvantages are the same as the payback period method discussed on p. 24, with the exception that this method does take into account the time value of money.

b. NET PRESENT VALUE (NPV)

The NPV method discounts cash flows over the life of an investment by using a measure of the cost of capital to evaluate the investment's profitability. If the resulting NPV is positive, then the investment would be accepted as profitable. If NPV equals zero, the cash inflows would only cover costs, so that the decision to accept the investment would be marginal (one would be indifferent about the accept/reject decision). If NPV is negative, then the decision would be to reject the investment.

$$(2)\ NPV = \sum_{t=1}^{n} \frac{C_t}{(1 + r)^t} - IO,$$

NPV = where C_t is the net cash flow at period t, r is the cost of capital, and IO is the initial outlay.

NPV is the strongest technique of capital budgeting for many reasons. First, it considers all cash flows of a project, the timing as well as the total amount of the cash flows generated. Second, the reinvestment rate or discount rate, assumed implicitly, is usually the cost of capital. This rate of return used to discount cash flows is a reasonable reinvestment rate, as it is assumed that a financial institution can, at a minimum, reinvest incoming cash flows in its own investments, using the cash flows as its own source of funds. (More will be discussed later concerning the measurement of the discount rate used in the NPV method.)

Third, NPV obeys the value additivity principle described previously and measures the contribution of a project to the wealth of shareholders, that is, a project with NPV equal to $1000 will increase shareholders' wealth by $1000. Hence, it is perfectly compatible with shareholders' welfare maximization (increasing the return to the owners of the bank).

If mutually exclusive projects are being compared, then the NPV measure will correctly select between them, given the appropriately selected discount rate.

Also, NPV avoids multiple rates of return, which can occur with other methods.

Finally, with NPV the analyst can use different discount rates over time to account for risks or inflation.

However, NPV does have disadvantages. The first disadvantage is NPV's dependence on the level of the discount rate and how it is determined. The higher the discount rate, the more likely the NPV of an investment will be negative, leading to a decision to reject

the investment. Therefore, it is important that an investment be analyzed carefully, using the correct discount rate.
Second, adjustments must be made when comparing investments with different expected times to maturity. As noted earlier, a three-year investment is not directly comparable to a ten-year investment, without some adjustment to account for the timing and number of cash flows. Third, if the same discount rate is used across time periods, this method assumes parallel shifts in a flat yield curve. In other words, the implicit assumption is made that interest rates are constant across all maturities and change by the same amount.[3]

1. ESTIMATING THE COST OF FUNDS

Financial institutions can raise funds from a variety of sources, depending on their size. A large commercial bank will be able to borrow from several sources:

- a variety of deposit-type accounts,
- the bond markets, and
- the equity markets through the issuance of new stock.

Smaller institutions may not have the same access to capital markets as large banks. In addition, many savings and loans and all credit unions are mutuals, that is, owned by the depositors, and cannot issue stock. Therefore, their sources of funds will be limited to deposit accounts and retained earnings.

The cost of funds can be estimated by summing all of the net expenses associated with deposits and other borrowings and dividing by total borrowed funds. This is an historical cost, and has the same disadvantages as the ratios analysis outlined on pp. 10-11. The simple average cost fails to identify the true economic impacts of current decisions. It is plausible only when the funding mix does not change and interest rates are stable over time. The best use of average cost of funds is in measuring past performance. However, these historical costs provide no insight into the future changes in interest costs.

According to classical theory, the cost of funds involved in lending decisions should be the marginal cost of funds, the cost of one additional dollar of investable funds. That one dollar can be from deposits raised or from equity capital. One way to estimate the marginal cost of funds is to assume that a bank finances new loans from a single source, e.g., 6-month Certificates of Deposit (CDs). The effective marginal cost then is equal to the CD rate adjusted for financial risk. This approach is especially useful when sources of funding can be identified. This method is used by financial institutions, but it ignores any costs of long-term debt and equity that an institution may incur. Therefore, it may give a biased estimate of cost of investable funds.

Sophistication can be added to generate multiple-pool costs of funds. The sources of funds are separated into several rough

categories with relatively homogeneous characteristics, such as volatility, liquidity, maturity, or interest sensitivity, but not as specific as a single source, such as 6-month CDs. The sources are allocated to uses, with each "pool's" being costed at its marginal rate. Different loans can be evaluated at different costs depending on relative weights of the sources of funds used to finance those loans. As a result, the borrowing costs of different sized firms can be evaluated based on the types of funds that they are able to raise. In pooling the various costs into a single measure, the weighted average cost of new funds can be calculated. This measure will reflect changes in the market interest rates. When market rates are expected to rise, the marginal cost of funds will be higher, thus leading to correct decisions in capital budgeting expenditures when using the NPV method.

An example of the problems of using marginal versus weighted-average costs of funds can be given by looking at a bank with a 20 percent cost of equity capital. With a required rate of return at this high a level, the bank could not afford to raise funds through an equity offering. However, if the bank wants to make new loans, it can borrow at one of the two costs of funds. If it uses the marginal approach, then the loans are funded at a more favorable rate on perhaps a 6-month CD at say 8 percent, as opposed to funding exclusively through raising new equity at 20 percent. (The bank can borrow through deposits, since deposits are insured and are not perceived as risky.) Assuming these are the bank's choices of raising funds, a weighted-average cost of funds would fall in between the two single-source funds, depending upon the percent of money raised from each source. In reality, if the bank funds the loans with CDs, then it is in essence expanding its assets and thus eroding its capital position. The bank will then not have sufficient capital to support new loans and will further add to the risk of shareholders and raise the cost of equity. In essence, this bank should shrink, not expand, its assets to reduce the cost of equity. If the weighted-average cost is used instead of the marginal cost, then the true cost of funds is known.

In the NPV analysis, however, the disadvantages must be understood. That is, every effort must be made to correctly estimate the cost of funds.

2. ESTIMATING THE DISCOUNT RATE

The bank manager must determine the cost of capital. The cost of capital acts as a discount rate in the NPV model on one hand and as a hurdle rate (the rate that investments must earn to at least cover costs) on the other. In previous discussion, we outlined the relative advantages and disadvantages of the NPV method over other methods of investment selection, given that reasonable estimates of cash flows are available. The NPV method indeed is used in the practice of financial institutions' credit decisions. To put the NPV model into use, however, we still must establish the proper

measure for the discount rate to use in the NPV equation. Therefore, it is important to look at the estimates that are used by financial institutions to satisfy the need for estimating the discount rate.

The discount rate used to measure the present value of cash flows is calculated in a variety of ways. There is no precise method that has emerged. A rule-of-thumb method is to take a standard measure of a risk-free rate, say the Treasury-bill rate, and add a premium to compensate for perceived risk. In using this method, the analyst must state what assumptions are being made in calculation of this discount rate. Generally, however, the bank's cost of capital (long-term debt and equity) is the standard measure, and using it in the NPV equation allows the bank to assume that cash flows can at least be reinvested at the rate at which the company borrows money to pay for the initial investment.

To help a bank more closely estimate the true cost of capital used to support an investment in student loans, an estimation of the weighted average cost of long-term borrowing and equity should be obtained. The CAPM model, as we will see, can give us estimates of the discount rate through the estimation of the required rate of return. However, we will also list the problems inherent in this analysis.

c. INTERNAL RATE OF RETURN (IRR)

A positive NPV implies an acceptable, profitable investment. To then measure what discount rate will cause the NPV to be equal to zero (the so-called breakeven rate of return), the IRR is measured. The IRR method takes the NPV calculation and solves for the discount rate that equates the discounted cash flows over the life of an investment to the initial cash outflow.

The IRR represents one discount rate that can be used in valuing an investment. One advantage of the IRR method is that the IRR is easy to compare to the cost of capital, since both are calculated in percentage rate terms and both can be used to determine the level of NPV. If the IRR is greater than the cost of capital, then the investment would be a profitable one and would be accepted. If the IRR equals the cost of capital, then the project would be considered marginal, as the costs of the investment would be covered, but there would be no "extra" profit to go to shareholders. With IRR less than the cost of capital, the investment would be rejected, as the investment would cost more than it could return.

In addition, different investments can be compared by simply comparing the IRRs of each to determine the returns of each. An investment that has an IRR of 30 percent can be compared with an investment with a return of say 20 percent. Strictly speaking, the one with the higher IRR will be chosen, since this indicates that

it is more profitable over a wider range of discount rates. However, as described below, the IRR does not consider the reinvestment rate, so that the simple comparison of IRRs can be faulty.

Other advantages are the same as for the NPV, since the methods use the same basic equation. However, the IRR has the added advantage that it does not rely on an estimate of the discount rate, since that is what is estimated.

Disadvantages are that the IRR may not always rank projects in the proper order, especially when different degrees of risk are being compared. If two projects are mutually exclusive, then IRR can give a wrong selection when compared with the NPV's selection. To continue our example above, the investment with an IRR of 30 percent is chosen based on the IRR method alone. However, if the discount rate (or cost of capital) used in the NPV calculation is lower than the IRR, then the decision to choose the IRR of 30 percent may be faulty. This difference results from assumptions about reinvestment rates for each project. NPV assumes reinvestment at the cost of capital, whereas IRR assumes reinvestment at the IRR. Therefore, for certain ranges of discount rates, the NPV technique of capital budgeting may choose the investment with the IRR of 20 percent, as it would give a higher profitability (NPV) within those ranges.

In addition, IRR implicitly assumes that cash flows can be reinvested at the IRR for each project. This assumption is faulty, since the IRR is the highest rate at which an investment is acceptable, and therefore, is an unlikely rate at which reinvestment can realistically and consistently take place.

The IRR technique does not obey the value additivity principle. A project might be selected by itself but excluded in combination with other projects. Management has to consider all possible combinations of projects and choose the combination that gives the greatest IRR.

If, over its life, an investment has cash outflows and inflows, then the IRR measure can give multiple or no solutions. An investment may require higher expenses in some periods than in others. Therefore, net cash flows for those periods may be negative. Thus, the investment may have any number of IRRs as solutions, depending on the cash flow series. The number of internal rates of return is limited to the number of reversals in signs of the cash flows. For example, given the cash flows:

Year 0	1	2
-$1600	$10,000	-$10,000

then we calculate two IRR values of 25 percent and 400 percent.

However, the number of IRRs can also be affected by the magnitudes of the cash flows. For example, given a cash flow stream of:

Year 0	1	2
-$1000	$1400	-$100

the calculated IRR is 32.5 percent only, despite the two sign reversals.[4]

Lastly, as described above, IRR does not take into account the sizes of competing investments. Given a 30 percent return on $10 million investment and a 100 percent return on a $1 investment, IRR would choose the $1 investment with the highest return. Therefore, IRR fails to give a sensible solution.

d. MODIFIED INTERNAL RATE OF RETURN (IRR*)

Because of the disadvantages of the multiple IRRs due to the possible negative cash flows, the IRR* measure was developed. The IRR* is a modification of the IRR and is a hybrid between the NPV and the IRR. It combines the NPV calculation using the estimated discount rate with the determination of the IRR as the rate at which discounted cash inflows equal outflows. The equation is more complicated, but it allows for a more accurate rate of return than IRR to be used in comparison with the cost of capital.

This technique compares the total net worth that an investment contributes to the bank by the end of its life to the initial investment. The ending net worth of an investment is calculated by taking the net benefits and compounding them at an appropriate reinvestment rate. By using the IRR* method, this value is then compared to the discounted cash outflows over the life of the investment plus the initial cost.

Advantages are similar to NPV. IRR* assumes the cost of capital is the reinvestment rate of cash flows over the life of the investment, and can vary as needed to account for differences in interest rates over time due to risk or inflation changes. Also, unlike the IRR method, there is only a single IRR* rather than multiple values, even for nonnormal cash flows.

As NPV, IRR* can choose among mutually exclusive investments. Given the choice between two similar investments but with different discount rates, IRR* can be used to choose the best one.

The disadvantages of IRR and IRR* are the same, as neither takes the sizes of investments into account and thus can fail to give a sensible answer.

e. RETURN TO NET WORTH

This method is similar to the IRR* method in that it compares the total net worth that an investment contributes to the bank by the end of its life to the initial investment. The ending net worth of an investment is calculated by taking the net benefits and compounding them at an appropriate reinvestment rate. However, by using the IRR method, this value is then compared to the initial investment, without concern to the negative cash flows during the life on the investment.

f. PROFITABILITY INDEX (PI)

This method measures the ratio of the discounted cash flows at the cost of capital to the cost of the initial investment. If the ratio is greater than one, then the project is accepted. If the ratio is equal to one, then costs are just being covered by cash inflows over the life of the investment, so the decision to accept is a marginal one. Of course, if the ratio is less than one, the investment is rejected.

$$(3)\ ProfitabilityIndex = \frac{\sum_{t=1}^{n} \frac{C_t}{(1+r)^t}}{IO}$$

PI is a benefit/cost ratio, similar to the NPV method, except that it measures, in terms of present value, the benefit generated by one unit of cost. However, due to the standardization process, it might lead to a decision that conflicts with value maximization. An investment with a larger NPV, but smaller PI, will be rejected by the PI rule.

D.) SUMMARY OF THE CAPITAL BUDGETING APPROACH

The NPV is the only capital budgeting method among competing capital budgeting methods that demonstrates all of the necessary characteristics outlined above (pp. 10-12): all cash flows, the opportunity cost of capital, and the principle of value additivity are all well accounted for. Also, the addition to shareholders' wealth can be appropriately evaluated. The investment with the highest NPV is the project that maximizes share price and the firm's value. The NPV also allows for adjustments to the riskiness of cash flows across time. In a world with periodic shocks, using a time-dependent, risk-adjusted discount rate is well justified. This method can be used alone or in combination with others, if desired. The choice is left to the analyst, with specific needs kept in mind.

The above capital budgeting methods have one thing in common. Each is investment specific. In determining profitability, various methods can be employed to adjust for uncertainties, but any adjustment is still investment specific, and does not determine the impact of increased uncertainty, or risk, on the bank. NPV can be adjusted for risk through cash flows, as well as through changes in the discount rate. This reflects an investment's added risk to the total risk of the institution, but other methods of measuring profitability can be used to measure this added risk to overall risk. These methods are discussed in the next sections. In fact, one of these methods, the Capital Asset Pricing Model, gives an estimate of NPV's risk-adjusted discount rate.

III. PORTFOLIO THEORY IN INVESTMENT VALUATION

In the preceding discussion, we assumed that cash flows of the investment are known with certainty. However, in the real world, the best we can do is to know the cash flows with some probability. For example, the cash flows of an investment may be $120 when the economy is good, $100 when the economy is normal, and $80 when the economy is poor, where the probabilities of the economy's being good, normal, or poor are 30 percent, 40 percent, and 30 percent, respectively. Here, the uncertainty of the cash flows are taken into account.

Two models are useful when there is uncertainty regarding future cash flows: Portfolio Theory and the Capital Asset Pricing Model (CAPM). To decide which model is more appropriate depends on the type of ownership of the company. For closely-held, privately-owned institutions, the portfolio theory may be more applicable. For widely-held, publicly-owned institutions whose shareholders are more capable of diversification, the CAPM is a better alternative.

In portfolio theory, any investment is characterized by its expected return and variance (or standard deviation) of return. The expected return is a measure of the "average" rate of return, and the variance is a measure of the risk of not achieving the expected return. In the above example, the expected cash flows will be equal to: ($120 * 0.3) + ($100 * 0.4) + ($80 * 0.3) = $100. The variance measures how far a cash flow deviates from the expected (or average) value. The standard deviation is the square root of the variance. One more needed definition is that of "covariance." The covariance measures the expected value of the product of the deviations (variances) of two cash flows from their respective means. This measure indicates how the deviations taken together differ from the mean and is used in the variance equation.

When more than one investment is held, the investor is said to hold a portfolio. To determine whether a portfolio is "good" or "bad" (too risky for the return received), investors can compare the expected return and standard deviation of the portfolio with other portfolios. Suppose there are two securities, 1 and 2, in a

portfolio. Security 1 has an expected return of 12 percent and represents 40 percent of the investment in the portfolio, and security 2 has an expected return of 10 percent and represents 60 percent of the portfolio. The expected return will equal 10.8 percent, calculated as above.

If the standard deviation of the two securities are 6 percent and 4 percent, respectively, and the covariance of returns on securities 1 and 2 is 0.2 percent, then the variance of the portfolio is equal to:

$$(4)\ variance = (0.4)(0.06) + (0.6)(0.04) + (2)(0.6)(0.002)$$
$$= 0.000576 + 0.000567 + 0.00096$$
$$= 0.00212$$

The standard deviation of the portfolio is the square root of the variance:

$$(5)\ standard\ deviation = \sqrt{0.00212} = 4.6\%.$$

These calculations say that the expected value of the return on the portfolio given the possible conditions in the economy is 10.8 percent, within a possible range of 10.8 percent +/- 4.6 percent. In other words, we can expect the return to be between 6.6 percent and 15.4 percent.[5]

Given investor preference for high returns and low risk, then if there is any other portfolio consisting of the same or different securities that 1.) has returns more than 10.8 percent, but with the same risk as measured by the standard deviation, 2.) has lower risk but the same expected return, or 3.) has the a higher return and a lower risk, then that portfolio is superior to the one calculated above.

The portfolio theory of capital budgeting is a useful model to account for uncertainty. In reality, the inclusion or acceptance of an investment generally will impact other investments, since diversification reduces risk. Similarly, for a bank's investing in student loans, the addition leads to diversification of the institution's portfolio and may attract more business for the institution. In addition, the inclusion will affect the return and risk structure of the institution as a whole. Portfolio theory is a good tool to evaluate these inputs.

Portfolio theory is not without disadvantages. It does take risk into account, but given the same risk level, it may erroneously select a 100 percent return on a $1 project rather than

10 percent on a $10 million project. That is, it considers

expected rates of returns only, but, like IRR and IRR*, it does not consider the size of the projects.

Next, the optimal weight of an acceptable project might be 1 percent within a portfolio, but a portfolio must be accepted as whole. If the acceptance of the investment inevitably increases the weight of the investment's weight in the portfolio, then the portfolio will be suboptimal. There will be a deviation from the best solution. For example, if a bank holds an optimal 1 percent of its loans in student loans and increases this percent, then the entire portfolio mix will be changed, so that the mix may no longer be optimal. The student loans may be profitable, but the portfolio taken as a whole may no longer be optimal.

Finally, when investors are able and willing to diversify their portfolio with low cost, the portfolio theory is not as appropriate. The standard deviation of the portfolio is the measurement of total risk (company-specific risk plus market risk). When more securities are added to the portfolio, the company-specific risk can be diversified away. Therefore, investors will not be compensated for bearing this diversifiable risk. Only market risk (risk that comes from general sources over which the bank has no control) will be considered. This is where the CAPM model is used.

IV.) CAPITAL ASSET PRICING MODEL IN PROJECT VALUATION

The CAPM approach of capital budgeting has a direct link to the NPV model. In section II, cash flows are assumed to be known with certainty and discounted at a constant rate, the cost of capital. Also, as discussed under uncertainty, the discount rate should be adjusted for the risk inherent in uncertain cash flows. The CAPM model makes the required adjustment.

When there is risk, a risk premium (or mark-up) can be added to the so-called risk-free rate. Of course, the premium is not arbitrary. It is determined by the amount of risk and the "price" of risk. The amount of risk is represented by market risk, the risk that cannot be diversified away by adding more securities. These risks include market factors such as economic changes or legislative changes, among others. The price of risk, the risk premium, is the difference between the expected return on the market as a whole and the risk-free rate.[6] The expected return on the market is a weighted average of expected returns on all assets, including stocks, bonds, real estate, coins, stamps, antiques, etc.

In terms of an equation, the CAPM can be written as:

$$(6)\quad E(r_i) = r_f + b_i[E(r_m) - r_f]$$

where:
$E(r_i)$ is the return on the ith investment,
r_f is the risk-free rate,
$E(r_m)$ is the estimated return on the market, and
b is the beta coefficient that measures how $E(r_i)$ changes as the risk premium changes.

The CAPM approach of capital budgeting within the context of the NPV model can be written as:

$$(7)\quad NPV_i = \frac{E(C)}{[1+E(r_i)]}$$

where E(C) is the expected value of cash flows.

In a multi-period NPV model, where there are multiple risky cash flows, then:

$$(8)\quad NPV = \sum_{t=1}^{n} \frac{E(C_t)}{[1 + E(r_t)]^t}.$$

To put the CAPM to practical use, each component can be estimated. The expected market return can be proxied by the average return on one of the stock indexes, the S&P 500 or the NYSE index. The beta coefficient can be estimated from the company's historical data, or from data from a comparable firm. For example, the financial institution can use the beta of a firm that originates student loans only, as an estimate of market risk of a student loan portfolio, assuming such a firm can be found.[7] The risk-free rate can be proxied by using the Treasury bill rate.

In the multi-period NPV model, it is implicitly assumed that the required (expected) rates of return are constant throughout the life of the project. This requires that the risk-free rate, the risk premium, and the beta risk are not changing across time. If any one of these factors fluctuates across time, then there is added risk to the single-period version of the CAPM equation 6.

The CAPM itself requires a set of rather restrictive assumptions. For example, there are no taxes, no transactions costs, and no information deficiency. It is also assumed that investors have the same expectations. Despite such restrictive assumptions, the CAPM indeed sustains empirical tests fairly well.

It has the advantages of being intuitive and simple to use.

The major problems with the CAPM's application in capital budgeting are the estimation and determination of the beta coefficient, the asset's expected life, the growth trends in the cash flows, and the way that investors process information and form forecasts. All have impacts on the beta value. For example, there are at least two ways of estimating beta. One is to use the historical stock price data from the bank itself, assuming that the bank is a stock-held company. There are problems with this method, however, in that the data is historic so that future decisions are based on past effects. Also, the bank's beta cannot accurately represent the risk of a new investment that has not before been included in the calculation. Therefore, assuming that the data is available, the second method, that of using the beta of a comparable bank that is solely in the one product area, is a more accurate one, assuming the data is available.

Management has to be careful in choosing the input for estimating beta. When a comparable bank is used, the institution must actually have the same beta as the investment under consideration. This kind of "pure play" is both time-consuming and expensive.

Therefore, the CAPM can theoretically give an accurate, risk-adjusted measure of the discount rate. However, in practice, there are problems in estimating the component parts.

V.) OPTION PRICING MODEL

The NPV technique of capital budgeting uses a present value factor to discount cash flows. Its application requires the determination of the discount rate, or cost of capital, as discussed above. The option pricing model indicates an alternative of estimating the present value factor, called the "time-state price."

The essence of the time-state price is in estimating how much investors should be compensated now to make them indifferent between receiving an amount now or another amount in the future, when there is certainty in the economy. For example, an investor will be indifferent to having $0.50 now and to having $1 next year when the economy is good. Then the $0.50 will be the time-state price while the economy is good next year. If we denote the time-state price as V_{st} and C_{st} as the time-state dependent cash flows, then the option pricing model approach to capital budgeting indicates that

$$(9)\quad NPV = \sum_{t} \sum_{s} V_{st}\, C_{st}.$$

The key point of the approach is in deriving the time-state price, V_{st}. Theoretically, investors' utility functions [6] have been used, but are not observable empirically. This requires estimation of the utility functions of investors, which is nearly impossible.

The emergence of the Black and Scholes (1973) option pricing model provides exciting solutions to this problem. In addition, Banz & Miller (1978) have shown how to estimate V_{st} from the Black & Scholes option pricing model. The model opens a new avenue in capital budgeting. However, since it is still being developed, it is too early to list the advantages and disadvantages of this model. Given that returns and risk of almost all projects depend on the state of the economy and on time, the option pricing model is a promising alternative method in capital budgeting.

VI. CONCLUSION

The NPV method of capital budgeting is theoretically and practically the best method for estimating student loan profitability. The NPV consistently leads to correct investment decisions using the necessary criteria, and data are generally available to financial institutions to estimate the component parts of the NPV equation. The method is not totally accurate due to its shortcomings, but the alternative methods are even less accurate.

APPENDIX B

This appendix is in addition to appendix A and only serves to further develop the theoretical explanations of Portfolio Theory and the Capital Asset Pricing Model.

To theoretically take the uncertainty of cash flows into account explicitly and systematically, two models are useful: Portfolio Theory and the Capital Asset Pricing Model (CAPM). The appropriateness of using one model rather than the other hinges on the type of ownership of the institution. For closely-held, privately-owned institutions, the portfolio theory may be more relevant. For widely-held, publicly-owned institutions, whose shareholders are assumed more capable of diversification, the CAPM is a better alternative.

I.) PORTFOLIO THEORY IN PROJECT VALUATION

As noted before, a portfolio is a set of securities held by an individual investor. The traditional portfolio theory is developed by Markowitz. The theory intends to address the problem of choosing a collection of marketable securities that have desirable characteristics with respect to return and risk. It is assumed that investors prefer higher expected returns to less and prefer less risk to more. Here, expected returns are measured by mathematical expected values, or the mean of all possible returns. Risks are measured with variance (or standard deviation) of returns. In general, returns are assumed normally distributed. What matters to investors is the mean and variance of return. Higher moments (skewness and kurtosis[9]) of the return distribution do not affect investment decisions. However, the normality assumption can be relaxed to include all stable symmetric (Paretian) distributions with which location and dispersion parameters rather than mean and variance are of concern.

When investors hold only one investment (or asset) in a single-asset portfolio, mean and variance of the returns of that investment are important. When more than one investment is held, it is the mean return and the variance of returns of the entire portfolio that the investment decision depends on. The portfolio mean return r_p is the weighted average of expected returns on securities in that portfolio, i.e.,

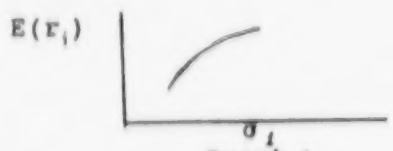
$$(1A)\quad E(r_p) = \sum_{j=1}^{n} X_j\, E(r_j)$$

where X_j is the proportion of security j in the portfolio, and $E(r_j)$ is the expected return of security j. The variance of portfolio return is given by

$$(2A)\quad s^2 = \sum_i \sum_j X_i X_j s_{ij}$$

where s_{ij} is the covariance of security i and j, and s_{ii} is defined as the variance of returns on security i. As long as the correlation coefficient between r_i and r_j is not equal to one, the inclusion of a security always reduces the variance of portfolio's return.

When different portfolios are compared, the one that offers any one of the following will be accepted: (1) a greater expected return for the same level of risk; (2) less risk and the same level of expected return; or (3) a smaller degree of risk and a greater expected return. Portfolios, which are selected in this manner, are called efficient portfolios. As all possible investments are considered and all possible combinations, the efficient frontier of all possible, "best" portfolios can be formed in an efficient frontier, shown in graph 1. This graph shows that at points below the efficient frontier, an investor could form a better portfolio, given the above criteria.

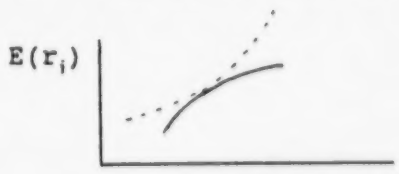


Graph 1

To put the Markowitz portfolio theory into the selection of the optimal investment project, three kinds of inputs are necessary: (1) the expected return on each project under evaluation; (2) the variance of the returns on each project, and (3) the correlation coefficient between the returns on each pair of projects. However, the estimates of these inputs rely on an appropriate measure of rate of return. Given the shortcomings of accounting rates of return and the regular IRR mentioned above, the following measures may be plausible: (1) adjusted IRR*, (2) return based on the profitability index, i.e., PI-1 (the PI value minus 1), which may be called the "profit rate," or (3) the discount return derived for the NPV. The management can subjectively assess a probability distribution of possible IRR* or [PV(C)/PV(IO)-1]. The IRR* or "profit rate" can be related to the general state of the economy (peak, recession and recovery, for example) or some other key factors. The associated probability of states of the

economy can be assigned according to subjective belief.

In the portfolio theory, assets are assumed to be infinitely divisible, so the efficient set is a smooth curve in a return-standard deviation plane. Investors choose a portfolio on the efficient frontier to maximize their utility, as shown in graph 2. In capital budgeting, projects should be accepted completely or rejected. The efficient set would be a discontinuous set of points denoting the optimal portfolios (or project composite). The management then chooses projects from the efficient set in accordance with the target return level and an acceptable level of risk.



Graph 2

Since student loans are only one part of the business of financial institutions, these institutions have to take all other types of loans into account to decide what types of loans to include in their portfolios. When an institution decides to offer student loans, it has to consider the contribution of the loans and accompanying risks to its total rate of return. Assuming that the institution is value maximizing, if the inclusion of student loans renders the institution unable to attain its efficient frontier, whatever weight combinations it chooses, then the institution will not undertake the new loan business. If, on the other hand, the inclusion of the new loans enables the institution to reach a "good" combination (in terms of the efficient portfolio of expected returns and risk), then the addition of student loans should be accepted. In other words, portfolio theory of capital budgeting provides decision-makers with criteria of capital budgeting that allow for consideration within a context of a "portfolio" rather than a single investment.

The essence of the portfolio theory in capital budgeting is that the riskiness of a single investment can not be evaluated appropriately only by looking at the results of the isolated investment. A corporation can be treated as a portfolio ofinvestment projects. To evaluate the return and riskiness of a new asset for a corporation, one should examine its impact on the dispersion of possible outcomes for the corporation as a whole. Just like investment decisions in financial assets, the institution should take into account the nature of risk associated with the project and its correlation with the risks of other investments. Decision criteria provided by portfolio theory allow for explicit

and systematic treatment of the contribution of a project to the corporation as a whole. In the case of student loans, we can evaluate the impact of a certain account (professional school, for instance) to the student loan account as a whole. The student loan can further be evaluated as a subset of the loan portfolio of the institution.

The caveat is that the variance of return as a measure of risk is the concept of total risk. Total risk consists of two kinds of risk: diversifiable (or company-specific risk) and non-diversifiable (or systematic, market risk). In general, the portfolio risk will be smaller than the weighted average of the securities' standard deviation, because variations in the individual securities' returns will be offsetting to some degree in a portfolio. As a rule, the risk of a portfolio will be reduced as the number of securities in the portfolio increase. That is, company-specific risk can be diversified away by holding a variety of securities. When investors are not able or willing to diversify company-specific risk, they have to assume market, as well as company-specific, risk by holding the nondiversified portfolio. For a privately-held financial institution, where shareholders are reluctant to give up control of the institution, total risk is more relevant. The portfolio method of capital budgeting is hence useful. For publicly-held institutions, whose shareholders are assumed to be willing and able to diversify risk at low cost, total risk is not a good measure of risk. Here is where the CAPM comes into the picture.

II.) THE CAPITAL ASSET PRICING MODEL IN INVESTMENT VALUATION

The CAPM identifies the relationship between expected return and risk, and measures the required rate of return that a bank's investors would expect to receive if the company were to invest in an asset. It is constructed on the basis of portfolio theory. The aforementioned Markowitz efficient set becomes a straight line when a risk-free asset (an asset that has no default risk) is introduced. We further assume that investors have quadratic (concave) utility functions and homogeneous (common) expectations.[10] The two-fund separation will follow, e.g., investors hold only the risk-free asset and risky market portfolio. The market portfolio consists of all assets, including real estate, coin, stamps, etc. The weight of a certain asset depends on its relative share in market capitalization. In equilibrium, we have the CAPM as:

$$(3A)\quad E(r_i) = r_f + b_i(E(r_m) - r_f),$$

where $E(r_i)$ = the expected return of asset i;
r_f = the risk-free rate;
b = $cov(r_i, r_m)/var(r_m)$;
r_m = the market return rate.

In words, it says that the expected rate of return is equal to the risk-free rate (return on the risk-free asset) plus a measure of risk, beta, times a risk premium.

The beta risk is also called systematic risk. It is a measurement of risk that comes from general sources over which the bank has no control. These risks include "market factors" such as economic changes and legislative changes, among others. In a world where diversification cost is low, investors can easily diversify away corporate-specific risk (or unsystematic risk) by investing in a large number and a variety of assets. Examples of company-specific risk are financial leverage risk (risk due to the use of debt), and risk that is inherent in its investment projects. Diversification can reduce risk but not eliminate it. The risk of return that co-varies with the market (i.e., market risk) can not be reduced by adding more assets into the portfolio. The investors will be compensated for bearing nondiversifiable (systematic) risk, but not for bearing diversifiable risk.

In portfolio selection, the CAPM provides criteria of ranking portfolio performance, provided that the market portfolio is mean-variance efficient, i.e., on the efficient frontier. In capital budgeting, the CAPM can enhance the NPV method by giving an estimate of investors' required rate of return rate used to discount cash flows. CAPM does this by offering systematic guidelines for adjusting risky flows and risk-adjusted discount factors when the NPV method is used. Since the CAPM maintains that investors are willing and able to diversify at low cost, it is more relevant for the project investment decision for publicly-held corporations than for closely-held ones.

(A) SINGLE-PERIOD MODEL

Since the CAPM is by construction a one-period model, it is readily transferred to capital budgeting when an investment has only a one-period life and a single cash stream. Let C_j be the net cash flow of project j at time period t. The C_js are now random variables rather than known constants. The NPV of project j within the CAPM is straightforward:

$$(4A)\ NPV_i = \frac{E(C_j)}{(1+E(r_j))},$$
$$where\ E(r_j) = r_f + b_j(E(r_m) - r_f).$$

In practice, the risky net cash flow C's can be inferred from a subjectively assessed distribution. In other words, managers can predict future cash flows based on information from their company or from other comparative companies. For example, cash flows can be predicted for the first period and second periods as:

period 1	$120	$100	$80
probabilities (%)	20	60	20
period 2	$125	$115	$108
probabilities (%)	60	30	10

The risk-free rate is generally proxied by returns on Treasury securities of the same holding period as the project. The return on market portfolio can be approximated by returns on market indexes.

The beta coefficient is even more difficult to measure than above rates. Normally, a simple way to calculate b is to use the beta calculated from the returns on the common stock of the whole company. This measure can be faulty, however, when the analysis is to compute a return on a single asset, or group of similar assets. This method is only appropriate when project, or asset, risk is the same as the risk of the existing business of the company. When the value-additivity principle holds, the value of the bank, which consists of two projects, A and B, will equal the sum of present values of project A and project B, each evaluated at its own opportunity cost of capital. Therefore, it can be an incorrect premise to assume that the particular asset beta can be measured using the same information used in estimating the risk of the entire institution, especially if the bank as a whole holds a somewhat diverse group of investments.

Another way to calculate a project's beta is to estimate the beta from the company's historical data, if old, similar projects are available. For a new venture project, the management could try to find a comparable company that only invests in one type of asset. For example, if a financial institution wants to begin offering student loans, then it could try to find a lender that offers only student loans. Then, from the historical stock data or from accounting data of the existing firm, a beta could be estimated to be used in the CAPM equation of the new entrant to derive the required rate of return for the new asset.

However, these two methods suggest problems in the analysis,

since historical data must be used. In addition, it may be quite difficult to find a single-product bank with which to compare betas.

In considering a project, the management has to take financial risk into account. Equation (3A) is true only if the project is financed by equity only. As long as the project is financed by debt, the increase of debt always increases the riskiness of the project, and, therefore, the opportunity cost of capital of the project. Given the beta of common stocks of the comparable company b^c, we get an unlevered b as:

$$(5A)\quad b^u = \frac{(b^c)}{[1 + (1-t)(\frac{D^c}{E^c})]}$$

where t is corporate tax rate, and D^c/E^c is the debt-equity ratio of the comparable company. Assuming that debt is risk-free, we can derive levered b as:

$$(6A)\quad b^l = b^u[1 + (1-t)(\frac{D}{E})],$$

where D/E is the debt-equity ratio of "our" company. Using the CAPM, the required rate of return on equity can be shown as follows:

$$(7A)\quad r_e = E(r_j) = r_f = b_l[E(r_m) - r_l].$$

The next step is to derive the weighted average cost of capital of the project as:

$$(8A)\quad WACC = W_e r_e + W_d r_d,$$

where W_e and W_d are weights of equity and debt, respectively, and r_d is the cost of debt, which is usually observable.

The NPV of a single cash flow investment is now in order:

$$(9A)\quad NPV = \frac{E(C)}{(1 + WACC)}.$$

The weighted average cost of capital (WACC) takes the change of business and financial risk into account. It hence can be used as a reasonable criteria of investment selection. The analysis can be easily expanded to include risky debt (for example Conine (1980)).

An alternative is to make adjustments to the risky cash flows, then discount at the risk-free rate. This is called the certainty-equivalent approach. The certainty equivalent is the amount of cash to be received for certain at time t for which the investor would be indifferent between having that cash or receiving the uncertain payment C. Using the CAPM, the certainty-equivalent is:

$$E(C_j) - \lambda\, cov(C_j, r_m),$$
$$where\ \lambda = \frac{[E(r_m) - r_f]}{\sigma^2},$$

the market risk premium. The NPV is the present value of certainty equivalent:

$$(10A)\ NPV = \frac{[E(C_j) - \lambda(C_j, r_m)]}{(1 + r_f)},$$

(B) MULTIPLE PERIOD MODEL

For investments with multiple cash flows in different periods, a multi-period model is needed, including any loans that have a maturity of more than one year. In the standard capital budgeting practice, we see that the following formula has been used:

$$(11A)\ NPV = \sum_{t=0}^{n} \frac{E(C_{jt})}{[1 + E(r_j)]^t}$$

The utilization of this formula in a multi-period uncertainty context is justifiable if the expected rate of return is constant over time. In other words, it is required that (1) the risk-free rate r_f , (2) the market risk premium, and (3) systematic risk of

the project, b, remain constant over the life of the project. A more general model by Haley and Schall can be formulated in which only r_f and the market risk premium are assumed constant.

If there is multi-period uncertainty, two types of risks need to be priced: (1) the basic risk of cash flow in each period measured by its relative covariance; and (2) risk due to potential revision of expectation regarding the expected value of the distribution. As a result, the discount factor r_t can be adjusted to the change of the market premium and the risk-free rate across periods accordingly. As before, the riskiness implied in debt-financed projects can be taken into account in the same manner as in the previous section.

(C) LIMITATIONS

The CAPM does provide a simple valuation model in the practice of capital budgeting. Some of the assumptions might be too restrictive, for example, a complete capital market and homogeneous expectations. But whether a theory can explain reality rather than the reasonableness of assumptions then justifies a theory. The real problem of the CAPM's application in capital budgeting lies in the determination and estimation of the beta coefficient. The following factors all have impacts on the value of b, to mention a few, (1) asset life, (2) the growth trends in the cash flows, (3) the pattern of expected cash flows over time, (4) the relationship between cash flows forecast errors and forecast errors for the market portfolio, and (5) the way that investors process information and form forecasts. Each of these factors needs careful estimation and affects the beta, accordingly.

It is necessary for the management to be cautious in choosing the input for estimating the beta. When a comparable company is used, the bank must actually have the same beta as the project under consideration. They must display the same magnitude and pattern on asset life, growth, timing, and distribution of expected cash flows over time, and the relative contribution of the characteristics to the value of the firm.

III.) OPTION PRICING MODEL

An option is a contract giving the holder the right to buy or sell an asset for a predetermined price. The most common options are options on common stocks. The buyers or holders of common stock call (put) options are given the right to buy (sell) underlying common stocks at the contract, or strike, price before or at expiration date. The famous Black-Scholes option pricing model is a basic model of valuing European call options, which can be exercised at the expiration date. There has been much literature that expands the Black-Scholes model to determine the value of put option and American options, which can be exercised at any time before expiration date.

The option pricing model can be applied to corporate financial decisions. With some modification, the model can be applied to a wide variety of financial instruments, such as convertible debt, underwriting contracts, loan commitments, insurance contracts, and others, since the essence of the option pricing model is to determine the value of contingent claims, i.e., the claim against a certain asset contingent on the state of nature. This is exactly in accordance with the properties of investment project with uncertain cash flows.

In a risk-adjusted discounting approach of capital budgeting, we use a risk-adjusted discount factor r to discount uncertain future cash flows. This procedure is similar to the so-called "time-state" model. The time-state model is a way of describing the existence and valuation of state-contingent consumption claims, i.e., it notes that future pay-off depends on states of the world (for instance, rainy/sunny, boom/recession) and have current prices in the market. The current prices of contingent claims depend on the marginal rate of substitution between consumption in different states and at different times. The application of the time-state model requires that one estimate the time-state versus the present price that the investor/consumer is willing to pay for a payoff of $1 in time t if state S occurs and zero otherwise. The estimation involves the assessment of an investor's utility function, which is not observable. This difficulty underlines the applicability of the time-state model, even though it is an extremely powerful theoretical tool.

The development of the option pricing model provides an avenue for the application of time-state model. Banz and Miller show how to derive time-state price V_{st} from the Black-Scholes model for both single and multiple period cases. Given the time-state price V_{st} and time-state net cash flow Z^{st}, the net present value of the project is:

$$(12A)\ NPV = \sum_{s=1}^{M} \sum_{t=1}^{T} V_{st} Z_{st}$$

where M is the number of state and T is number of time period.

To apply the option pricing model to student loans, the state price (or to put it another way, the present value factor) can be derived from the Black-Scholes option pricing model. The state price, V_{st} shows the present value of $1 in period t when state s occurs, for instance, the economy is expected to be poor three years from now. Managers can predict cash flows considering all possible states of the economy for the lives of the student loans.

The option pricing model offers an alternative way to estimate the present value factor. It is not as intuitive as the CAPM model and the calculations are more complicated. It can, therefore, be used as a compliment to the usual NPV with the CAPM model of capital budgeting.

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2. In finance, the interest rate has many names, according to its use.

3. An analyst can use different discount rates across time periods, if rate projections are available, by simply calculating the NPV based on cash flows discounted at the discount rate applicable to that time period. The difference between this and the first method is that the first method uses the summation technique across time periods that are discounted at the same rate, whereas this second method has NPV summed over individual periods.

4. "We note that the equation for solving for the internal rate of return . . . is an nth degree polynomial, having n years in which cash flows occur. Therefore, the formula has n roots. For conventional proposals, only one of these roots is a real number and n-1 roots are imaginary. As a result, the proposal is said to have a unique internal rate of return. In other words, the net-present value line... crosses the zero line only once. With a nonconventional investment proposal (one with reversals in signs of the cash flows) . . . more than one of the roots is a real number, and the present-value line crosses the zero line more than once." Van Horne, James C., Financial Management and Policy, 5th Ed., Prentice Hall, 1980, p. 137.

5. Actually, the range as given here is within one standard deviation, which gives a 65 percent possibility that the true return falls within that range. If two or three standard deviations are used, then one could be almost 95 percent and 99 percent, respectively, sure that the true return fell within those ranges.

6. The risk-free rate is usually represented by the Treasury bill rate of the comparable maturity to the investment being measured.

7. The beta from a comparable bank cannot be used directly in the CAPM formula. Adjustments for financial leverage (debt and equity ratio) of the comparative bank as well as of our bank have to be made. Interested readers are referred to the appendix for the adjustments.

8. Utility functions are used in economics to describe the preferences of individuals and represent different combinations of goods as they give the same level of satisfaction.

9. Skewness and kurtosis refer to statistical measures of distribution. Skewness demonstrates a lack of symmetry in a frequency distribution. Kurtosis refers to the distribution around

the mean, as to the peakness or flatness of the curve on a graph. It refers to the concentration of values about the mean.

10. An example of the quadratic utility function would be :

$$U(R) = aR + bR^2 ,$$

where R is a rate of return, and a and b are constants.